

© International Baccalaureate Organization 2026

All rights reserved. No part of this product may be reproduced in any form or by any electronic or mechanical means, including information storage and retrieval systems, without the prior written permission from the IB. Additionally, the license tied with this product prohibits use of any selected files or extracts from this product. Use by third parties, including but not limited to publishers, private teachers, tutoring or study services, preparatory schools, vendors operating curriculum mapping services or teacher resource digital platforms and app developers, whether fee-covered or not, is prohibited and is a criminal offense.

More information on how to request written permission in the form of a license can be obtained from <https://ibo.org/become-an-ib-school/ib-publishing/licensing/applying-for-a-license/>.

© Organisation du Baccalauréat International 2026

Tous droits réservés. Aucune partie de ce produit ne peut être reproduite sous quelque forme ni par quelque moyen que ce soit, électronique ou mécanique, y compris des systèmes de stockage et de récupération d'informations, sans l'autorisation écrite préalable de l'IB. De plus, la licence associée à ce produit interdit toute utilisation de tout fichier ou extrait sélectionné dans ce produit. L'utilisation par des tiers, y compris, sans toutefois s'y limiter, des éditeurs, des professeurs particuliers, des services de tutorat ou d'aide aux études, des établissements de préparation à l'enseignement supérieur, des fournisseurs de services de planification des programmes d'études, des gestionnaires de plateformes pédagogiques en ligne, et des développeurs d'applications, moyennant paiement ou non, est interdite et constitue une infraction pénale.

Pour plus d'informations sur la procédure à suivre pour obtenir une autorisation écrite sous la forme d'une licence, rendez-vous à l'adresse <https://ibo.org/become-an-ib-school/ib-publishing/licensing/applying-for-a-license/>.

© Organización del Bachillerato Internacional, 2026

Todos los derechos reservados. No se podrá reproducir ninguna parte de este producto de ninguna forma ni por ningún medio electrónico o mecánico, incluidos los sistemas de almacenamiento y recuperación de información, sin la previa autorización por escrito del IB. Además, la licencia vinculada a este producto prohíbe el uso de todo archivo o fragmento seleccionado de este producto. El uso por parte de terceros —lo que incluye, a título enunciativo, editoriales, profesores particulares, servicios de apoyo académico o ayuda para el estudio, colegios preparatorios, desarrolladores de aplicaciones y entidades que presten servicios de planificación curricular u ofrezcan recursos para docentes mediante plataformas digitales—, ya sea incluido en tasas o no, está prohibido y constituye un delito.

En este enlace encontrará más información sobre cómo solicitar una autorización por escrito en forma de licencia: <https://ibo.org/become-an-ib-school/ib-publishing/licensing/applying-for-a-license/>.

Mathematics: analysis and approaches

Higher level

Paper 3

20 May 2026

Zone A afternoon | **Zone B** afternoon | **Zone C** afternoon

1 hour 15 minutes

Instructions to students

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[55 marks]**.

Answer **all** questions in the answer booklet provided. Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 26]

This question asks you to explore a game of chance.

A game of chance involves attempting to win sweets from a basket through a series of rounds. The rules of the game are:

- The player starts the game with at least one sweet.
- At the start of each round the player spins a spinner.
- If the spinner shows “win”, the player wins a sweet from the basket; otherwise, they must return one sweet to the basket.
- The game is played until either all the sweets are won, or the player has no sweets remaining.

Initially, consider the case where there is a total of three sweets in the game.

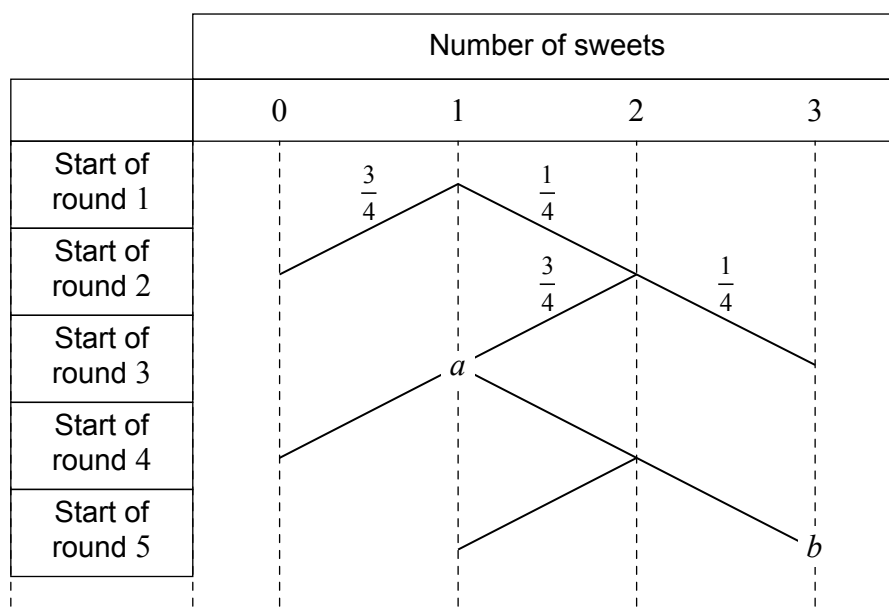
Fiona starts the game with one sweet and there are two sweets in the basket to win.

The probability of obtaining a “win” in any round is $\frac{1}{4}$, independent of any other spin.

(This question continues on the following page)

(Question 1 continued)

The following diagram shows the possible number of sweets that Fiona has at the start of each round, and the possible outcomes of the initial rounds.



If Fiona wins round 1 she will have exactly two sweets at the start of round 2.

If she loses round 1, she will have no sweets remaining at the start of round 2 and the game ends.

The probability of Fiona having exactly one sweet at the start of round 3 is a , and the probability of her having three sweets at the start of round 5 is b .

(a) (i) Show that $a = \frac{3}{16}$. [1]

(ii) Find the value of b . [2]

At the start of round 5, Fiona has exactly one sweet.

(b) Write down the probability that she will still have one sweet after two more rounds. [2]

(This question continues on the following page)

(Question 1 continued)

Now consider the case where there is a total of w sweets in the game.

Let R_n denote the probability that Fiona will eventually have all w sweets, when she starts a round with n sweets.

If Fiona finishes a round with no sweets, $n = 0$, the probability that she will eventually have all the sweets, R_0 , is zero and the game ends.

For $n > 0$, if Fiona wins the next round, she will have $n + 1$ sweets and the probability of her eventually having all w sweets is now R_{n+1} .

Similarly, if Fiona loses the next round, she will have $n - 1$ sweets and the probability of her eventually having all w sweets is now R_{n-1} .

The probability that the spinner shows “win” is now p , where $0 < p < 1$, and the probability that it shows “not win” is q , where $q = 1 - p$ and $p \neq q$.

At the start of each round, R_n is given by

$$R_n = qR_{n-1} + pR_{n+1}, \text{ where } n > 0.$$

Consider the case where $w = 4$ and $p = \frac{1}{4}$.

(c) Write down an expression for R_2 in terms of R_1 and R_3 . [1]

(d) It is known that when Fiona starts a round with exactly one sweet, the probability that she will win the game is $R_1 = 0.025$. When she starts a round with exactly three sweets, the probability that she will win the game is $R_3 = 0.325$.

Find the value of R_2 . [2]

(This question continues on the following page)

(Question 1 continued)

For the following parts of this question, consider the case where there is a total of w sweets in the game and the probability that the spinner shows “win” is p .

(e) By expressing R_n as $(p + q)R_n$, show that $R_{n+1} - R_n = \frac{q}{p}(R_n - R_{n-1})$. [2]

(f) (i) Hence, find an expression for $R_2 - R_1$ in terms of p , q and R_1 . [1]

(ii) Show that $R_3 - R_2 = \left(\frac{q}{p}\right)^2 R_1$. [2]

(iii) Deduce an expression for $R_n - R_{n-1}$ in terms of n , p , q and R_1 . [1]

(g) (i) By considering $(R_2 - R_1) + (R_3 - R_2) + (R_4 - R_3) + \dots + (R_n - R_{n-1})$, or otherwise, show that

$$R_n = R_1 \left(1 + \frac{q}{p} + \left(\frac{q}{p}\right)^2 + \dots + \left(\frac{q}{p}\right)^{n-1} \right). \quad [4]$$

(ii) Hence, given that $R_1 = \frac{1 - \frac{q}{p}}{1 - \left(\frac{q}{p}\right)^w}$, show that $R_n = \frac{1 - \left(\frac{q}{p}\right)^n}{1 - \left(\frac{q}{p}\right)^w}$. [4]

There is now a large number of sweets in the game.

Fiona starts a game with m sweets, where $p = 0.495$.

Once she has obtained 10 additional sweets, she will have all the sweets and the game ends.

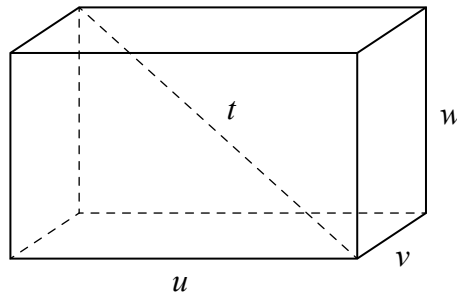
(h) Determine the minimum value of m for Fiona to have at least an 80% chance of winning 10 additional sweets. [4]

2. [Maximum marks: 29]

This question asks you to explore the relationship between complex numbers and a cuboid whose dimensions and interior diagonal are all integer lengths. This cuboid is called a Pythagorean box.

A Pythagorean box, denoted as $PB(u, v, w, t)$, is a cuboid with dimensions $u \times v \times w$ and an interior diagonal of length t , where $u, v, w, t \in \mathbb{Z}^+$. This is shown in the following diagram.

diagram not to scale



- (a) For the case where $u = 9$, $v = 2$ and $w = 6$, find the value of t . [2]

The complex numbers z_1 and z_2 are used to generate a Pythagorean box, $PB(u, v, w, t)$, if

$$u = 2\operatorname{Re}(z_1 z_2^*), \quad v = 2\operatorname{Im}(z_1 z_2^*), \quad w = |z_1|^2 - |z_2|^2 \quad \text{and} \quad t = |z_1|^2 + |z_2|^2, \quad \text{where } u, v, w, t \in \mathbb{Z}^+.$$

Conversely, for any Pythagorean box, $PB(u, v, w, t)$, there exist two complex numbers, z_1 and z_2 , which satisfy u, v, w and t as defined above.

Consider the two complex numbers $z_1 = a + bi$ and $z_2 = c + di$, where $a, b, c, d \in \mathbb{Z}^+$.

- (b) (i) Find an expression for $z_1 z_2^*$ in terms of a, b, c and d . [2]
 (ii) Hence, find an expression for v in terms of a, b, c and d . [1]
- (c) Consider the case where $z_1 = 3 + 7i$ and $z_2 = c + 6i$ are used to generate a Pythagorean box.
 (i) Show that $v = 14c - 36$. [1]
 (ii) Show that the only possible values of c are 3 and 4. [4]
 (iii) For $c = 4$, find the values of u, v, w and t . [5]

(This question continues on the following page)

(Question 2 continued)

There are three Pythagorean boxes with an interior diagonal of length 23.

- (d) Given that $23 = 1^2 + 2^2 + 3^2 + 3^2$, determine the dimensions of **one** of these Pythagorean boxes. [6]

The dimensions and interior diagonal of the Pythagorean box, $PB(u, v, w, t)$, can be used to form a complex number, z , such that $z = \frac{u + vi}{t - w}$.

Consider the Pythagorean box $PB(26, 38, 43, 63)$.

- (e) Show that, for this Pythagorean box, $z = \frac{13 + 19i}{10}$. [1]

The complex number z can also be written as $\frac{z_1}{z_2}$, where z_1 and z_2 generate the Pythagorean box $PB(26, 38, 43, 63)$.

It is given that $\frac{z_1}{z_2}$ can be expressed as $\frac{z_1 z_2^*}{|z_2|^2}$.

- (f) (i) By considering $t - w$, show that $c^2 + d^2 = 10$. [2]
 (ii) Hence, given that $c > d$, find z_1 and z_2 . [5]
-

Disclaimer:

Content used in IB assessments is often taken from authentic, third-party sources. The views expressed within them belong to their individual authors and/or publishers and do not necessarily reflect the views of the IB. Sometimes fictitious companies, products or individuals are included in assessments; any similarities with actual entities are purely coincidental. Any trademarks™ or registered® trademarks included are used for illustrative purposes only, and use does not imply any affiliation with or endorsement by the International Baccalaureate.